Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Paper reference **8FM0/26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1.

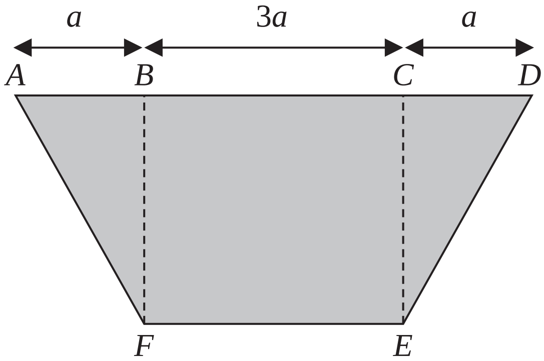

Figure 1

A uniform plane lamina is in the shape of an isosceles trapezium *ABCDEF*, as shown shaded in Figure 1.

- *BCEF* is a square
- $AB = CD = a$
- $BC = 3a$

(a) Show that the distance of the centre of mass of the lamina from *AD* is $\frac{11a}{8}$

(5)

The mass of the lamina is *M*

The lamina is suspended by two light vertical strings, one attached to the lamina at *A* and the other attached to the lamina at *F*

The lamina hangs freely in equilibrium, with *BF* horizontal.

(b) Find, in terms of *M* and *g*, the tension in the string attached at *A*

(2)

(a)

	ABF	BCEF	CDE	Lamina
Mass	$\frac{3}{2}a^2$	$9a^2$	$\frac{3}{2}a^2$	$12a^2$ (1)
Distance	a	$\frac{3}{2}a$	a	$\bar{y}$ (1) ← vertical distance from AD

(1) AD: $\frac{3}{2}a^2 \times a + 9a^2 \times \frac{3}{2}a + \frac{3}{2}a^2 \times a = 12a^2 \times \bar{y}$ (1)

↑ moments about AD

$\frac{33}{2}a^3 = 12a^2\bar{y}$ $\div a^2$

$\frac{33}{2}a = 12\bar{y}$

$\frac{11a}{8} = \bar{y}$ (1)

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Question 1 continued

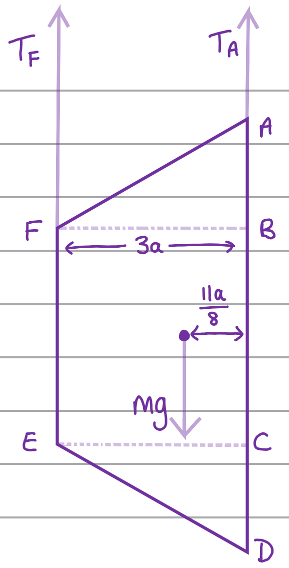
(horizontal distance from F)

(b) $\overset{\curvearrowright}{F}$: $mg \times \left(3a - \frac{11a}{8}\right) = 3aT_A$ (1)

$$\frac{13}{8}mga = 3aT_A$$

$$\frac{13}{8}mg = 3T_A$$

$$\frac{13}{24}mg = T_A$$ (1)



Question 1 continued

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Question 1 continued

(Total for Question 1 is 7 marks)

2.

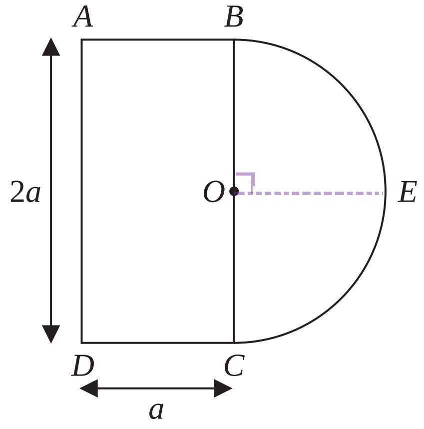


Figure 2

Uniform wire is used to form the framework shown in Figure 2.

In the framework

- $ABCD$ is a rectangle with $AD = 2a$ and $DC = a$
- BEC is a semicircular arc of radius a and centre O, where O lies on BC

The diameter of the semicircle is BC and the point E is such that OE is perpendicular to BC.

The points A, B, C, D and E all lie in the same plane.

(a) Show that the distance of the centre of mass of the framework from BC is

$$\frac{a}{6+\pi}$$

(5)

The framework is freely suspended from A and hangs in equilibrium with AE at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ.

(4)

The mass of the framework is M.

A particle of mass kM is attached to the framework at B.

The centre of mass of the loaded framework lies on OA.

(c) Find the value of k.

(3)

Question 2 continued

(a)

	ABCD	:	BEC	.	framework	
Mass	$6a$		πa		$6a + \pi a$	(1)
Distance	$\frac{1}{2}a$		$\frac{2a}{\pi}$		$\bar{x}$	(1)

$\leftarrow 2\pi = \bigcirc = 360°$

$\pi = D = 180°$

$\uparrow$

radius r, angle at centre 2α : $\frac{r\sin\alpha}{\alpha}$ from centre (formula book)

$2\alpha = \pi \Rightarrow \alpha = \frac{\pi}{2}$ $\quad \frac{a\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{2a}{\pi}$

on either side of BC

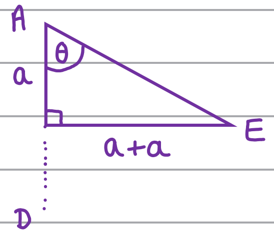
(1) $\curvearrowleft$ BC . $6a \times \frac{1}{2}a - \pi a \times \frac{2a}{\pi} = (6a + \pi a)\bar{x}$ (1)

$3a^2 - 2a^2 = a\bar{x}(6 + \pi)$ $\quad \div a$

$a = \bar{x}(6 + \pi)$

$\frac{a}{6 + \pi} = \bar{x}$ (1)

(b)



$\angle DAE$. $\tan\theta_1 = \frac{2a}{a} = 2$ $\leftarrow \tan\theta = \frac{O}{A}$

$\theta_1 = \tan^{-1}(2)$ (1)

Let G = centre of mass

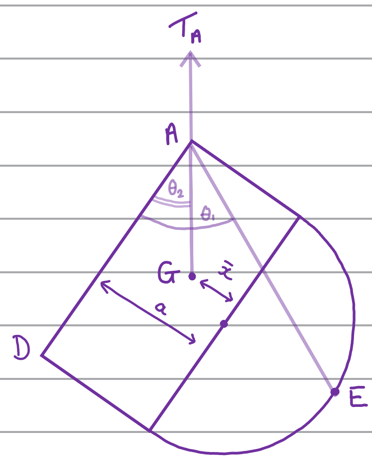
rationalise $\downarrow$

$\angle DAG$. $\tan\theta_2 = \frac{a - \frac{a}{\pi + 6}}{a} = \frac{5 + \pi}{6 + \pi}$ (1)

$\angle GAE = \angle DAE - \angle DAG$ (1)

$= \tan^{-1}(2) - \tan^{-1}\left(\frac{5+\pi}{6+\pi}\right)$

$= 21.7°$ (3 s f) (1)



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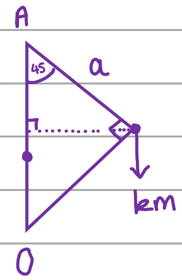
Turn over

Question 2 continued

(c) If G lies on OA, then it hangs directly below A, and $\angle OAB = 45°$



Distance OA → B = $a\sin 45°$ ← $\sin\theta = \frac{O}{H}$

Moments about OA. (1) $kM \times a\sin 45 = M\bar{x}\sin 45$ (1)

÷ $M\sin 45$

$ka = \frac{a}{6+\pi}$

÷ a

$k = \frac{1}{6+\pi}$

$k = 0.109$ (1) (3 s f)

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Question 2 continued

(Total for Question 2 is 12 marks)

3. A cyclist is travelling around a circular track which is banked at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

The cyclist moves with constant speed in a horizontal circle of radius r.

In an initial model,

- the cyclist and her cycle are modelled as a particle
- the track is modelled as being rough so that there is sideways friction between the tyres of the cycle and the track, with coefficient of friction μ, where $\mu < \frac{4}{3}$

Using this model, the maximum speed that the cyclist can travel around the track in a horizontal circle of radius r, without slipping sideways, is V.

(a) Show that $V = \sqrt{\frac{(3 + 4\mu)rg}{4 - 3\mu}}$

(7)

In a new simplified model,

- the cyclist and her cycle are modelled as a particle
- the motion is now modelled so that there is **no** sideways friction between the tyres of the cycle and the track

Using this new model, the speed that the cyclist can travel around the track in a horizontal circle of radius r, without slipping sideways, is U.

(b) Find U in terms of r and g.

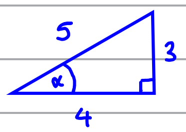
(2)

(c) Show that $U < V$.

(2)

(a) Resolve vertically: (1) $mg = R\cos\alpha - F\sin\alpha$ (1) (1)

Resolve horizontally (1) $R\sin\alpha + F\cos\alpha = m \times \frac{V^2}{r}$ (2) (1) ← $F = m \times a$, $a = \frac{V^2}{r}$

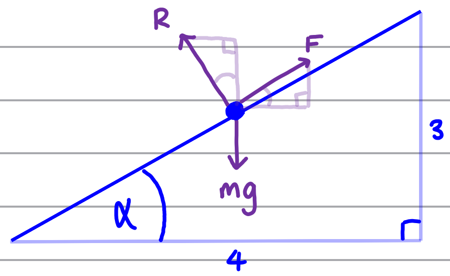


$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

(1) $mg = \left(\frac{4}{5} - \frac{3}{5}\mu\right)R$ (1) ← replace $F = \mu R$

(2). $\frac{mV^2}{r} = \left(\frac{3}{5} + \frac{4}{5}\mu\right)R$



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Question 3 continued

$mg = \left(\frac{4}{5} - \frac{3}{5}\mu\right)R \Rightarrow m = \frac{\left(\frac{4}{5} - \frac{3}{5}\mu\right)R}{g}$ sub. into ②

$\frac{\left(\frac{4}{5} - \frac{3}{5}\mu\right)R}{g} \times \frac{v^2}{r} = \left(\frac{3}{5} + \frac{4}{5}\mu\right)R$ ①

$\div R$

$\frac{\left(\frac{4}{5} - \frac{3}{5}\mu\right)v^2}{gr} = \frac{3}{5} + \frac{4}{5}\mu$

$\times 5$

$\frac{(4 - 3\mu)v^2}{gr} = 3 + 4\mu$

$\times gr$

$(4 - 3\mu)v^2 = (3 + 4\mu)gr$

$\div (4 - 3\mu)$

$v^2 = \frac{(3 + 4\mu)rg}{4 - 3\mu}$

$\sqrt[2]{\ }$

$v = \sqrt{\frac{(3 + 4\mu)rg}{4 - 3\mu}}$ ①

(b) No friction so $\mu = 0$. ①

$u = \sqrt{\frac{(3 + 4(0))rg}{4 - 3(0)}}$ sub into (a)

$u = \sqrt{\frac{3rg}{4}}$ ①

Question 3 continued

(c) In U, $\mu = 0$. $3 + 4\mu > 3$ and $4 - 3\mu < 4$ (1)

$\therefore \frac{3}{4} < \frac{3+4\mu}{4-3\mu}$ $\therefore U < V$ (1)

Question 3 continued

(Total for Question 3 is 11 marks)

4. A particle P moves on the x-axis. At time t seconds the velocity of P is $v\,\text{m s}^{-1}$ in the direction of x increasing, where

$$v = \frac{1}{2}(3e^{2t} - 1) \qquad t \geqslant 0$$

The acceleration of P at time t seconds is $a\,\text{m s}^{-2}$

(a) Show that $a = 2v + 1$

(2)

(b) Find the acceleration of P when $t = 0$

(1)

(c) Find the exact distance travelled by P in accelerating from a speed of $1\,\text{m s}^{-1}$ to a speed of $4\,\text{m s}^{-1}$

(7)

(a) $a = \frac{dv}{dt} = \frac{1}{2} \times 2 \times 3e^{2t} = 3e^{2t}$ ①

$\int dt \left(\begin{matrix} x \\ v \end{matrix} \right) \frac{d}{dt}$

$v = \frac{1}{2}(a-1)$ ← sub. $3e^{2t} = a$

$\int dt \left(\begin{matrix} \ \\ a \end{matrix} \right) \frac{d}{dt}$

$2v = a - 1$

①) rearrange to get $V =$

$2v + 1 = a$

(b) $a = 3e^{2t} = 3e^{2\times0} = 3 \times 1 = 3\,\text{ms}^{-2}$ ①

(c) $x = \int \frac{1}{2}(3e^{2t} - 1)\,dt$ ①

$x = \frac{1}{2}\left(\frac{3}{2}e^{2t} - t\right)$ ①

When $v = 1\,\text{ms}^{-1}$: $1 = \frac{1}{2}(3e^{2t} - 1)$ ①

$2 + 1 = 3e^{2t}$

$t = \frac{1}{2}\ln(1) \Rightarrow t = 0$ ①

At $t = 0$ $x_1 = \frac{1}{2}\left(\frac{3}{2}e^{2\times0} - 0\right) = \frac{3}{4}$

Question 4 continued

When $v = 4\text{ ms}^{-1}$: $4 = \frac{1}{2}(3e^{2t} - 1)$

$8 + 1 = 3e^{2t}$

$t = \frac{1}{2}\ln(3)$ (1)

At $t = \frac{1}{2}\ln(3)$: $x_2 = \frac{1}{2}\left(\frac{3}{2}e^{2\times\frac{1}{2}\ln 3} - \frac{1}{2}\ln 3\right)$

$x_2 = \frac{1}{2} \times \frac{3}{2} \times 3 - \frac{1}{4}\ln 3$

$x_2 = \frac{9}{4} - \frac{1}{4}\ln 3$

$x_2 - x_1 = \frac{9}{4} - \frac{1}{4}\ln 3 - \frac{3}{4} = \frac{6}{4} - \frac{1}{4}\ln 3$ (1)

$\therefore$ Distance $= \frac{3}{2} - \frac{1}{4}\ln 3$ m (1)

Question 4 continued

(Total for Question 4 is 10 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS